MATH 2 **EXAM #1** **NAME:** _____ key

Please Note: Use only **one side** of the provided blank papers. Do the problems in order. Use the same guidelines as in your homework assignments. Show **all your work** to backup your answers. Merely stating the answer of a problem results in no credit.

1. Solve each of the following equations:

a. $\dfrac{x+1}{x-2}=\dfrac{3}{x-2}-\dfrac{3}{x}$

b. $-2|3-x|+4=2$

2. Rewrite the following expression without the absolute value: $|3x-1|$

3. Solve each of the following inequalities, show the solution set on a number line and represent the solution set using the interval notation.

a. $-5\le 3-2x<0$

b. $2x-5<3x+1\le 4x+2$

c. $-2|3-2x|+4<-2$

d. $-2|x+2|\ge -2$

e. $|2-x|\le 0$

f. $|x+1|>-1$

g. $x^2-2x\le 8$

h. $\dfrac{-2x^2(x+1)^3}{(x-1)^4(x-2)}\le 0$

4. The following points are given: $A(2,-3)$ and $B(0,2)$

a. Find the length of the line segment $\overline{AB}$.

b. Find the midpoint of $\overline{AB}$.

c. Find the equation of a circle if the line segment $\overline{AB}$ is one of its diameters.

d. Find the equation of a line passing through points A and B.

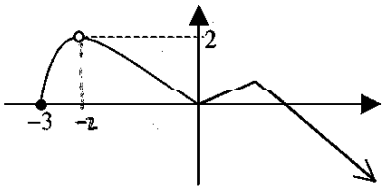
5. Find the center, radius and sketch the graph of the following circle: $2x^2+2y^2+8x+7=0$

6. Using the following graph,

a. state if it is a function.

b. find the domain.

c. find the range.



7. Sketch the graph of the following functions using the transformation concept that was presented in the lecture. Number each transformation and briefly state what each transformation is doing.

a. $y=\sqrt{x-1}$

b. $y=2x^3+1$

c. $y=-\sqrt{-x}$

d. $y=-2|x-1|+3$

8. (EXTRA CREDIT) Sketch the graph of $y=[\![x^2]\!]$

① (a) $\frac{x+1}{x-2} = \frac{3}{x-2} - \frac{3}{x}$

$(x-2)(x)\ \frac{x+1}{x-2} = x(x-2)\frac{3}{x-2} - x(x-2)\frac{3}{x}$; $x \neq 0,\ x \neq 2$

$x(x+1) = 3x - (x-2)3$ //

$x^2 + x = 3x - 3x + 6$ //

$x^2 + x - 6 = 0$ //

$(x+3)(x-2) = 0$ //

$x = -3$; $x = 2$ $\quad x \neq 0,\ x \neq 2$

contradiction

So, $x = 2$ is not acceptable.

(5)

(b) $-2|3-x|+4 = 2$

$-2|3-x| = -2$

$|3-x| = 1$

$3-x = 1$ OR $3-x = -1$

$x = 2$ $\quad x = 4$

(4)

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② $|3x-1| = \begin{cases} 3x-1 & \text{if } 3x-1 \geq 0 \\ -(3x-1) & \text{if } 3x-1 < 0 \end{cases}$

(4)

$\Rightarrow |3x-1| = \begin{cases} 3x-1 & \text{if } x \geq \frac{1}{3} \\ -3x+1 & \text{if } x < \frac{1}{3} \end{cases}$

(3) (a) $-5 \leq 3-2x < 0$

$-8 \leq -2x < -3$

$4 \geqslant x > \frac{3}{2}$

$\frac{3}{2} < x \leq 4$

Interval notation: $\left(\frac{3}{2}, 4\right]$

(5)

(b) $2x-5 < 3x+1 \leq 4x+2$

AND

$2x-5 < 3x+1$ AND $3x+1 \leq 4x+2$

$-x < 6$ AND $-x \leq 1$

$x > -6$ AND $x \geqslant -1$

Interval notation: $[-1, +\infty)$

(5)

(c) $-2|3-2x|+4 < -2$

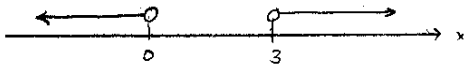
$-2|3-2x| < -6$

$|3-2x| > 3$

$3-2x < -3$ OR $3-2x > 3$

$-2x < -6$ $\quad -2x > 0$

$x > 3$ OR $x < 0$



Interval notation: $(-\infty, 0) \cup (3, +\infty)$

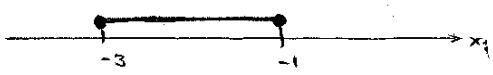
(5)

(d) $-2|x+2| \geqslant -2$

$|x+2| \leqslant 1$

$-1 \leqslant x+2 \leqslant 1$

$-3 \leqslant x \leqslant -1$



Interval Notation: $[-3, -1]$

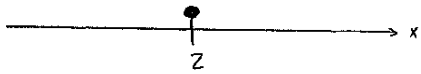
(5)

(e) $|2-x| \leqslant 0$

$0 \leqslant 2-x \leqslant 0$

$-2 \leqslant -x \leqslant -2$

$2 \leqslant x \leqslant 2 \Rightarrow x = 2$



Interval Notation: $\{2\}$

(4)

(f) $|x+1| > -1$

+ or zero $\Rightarrow$ $|x+1|$ is always greater than -1.

So, solution is: x can be any real number (4)

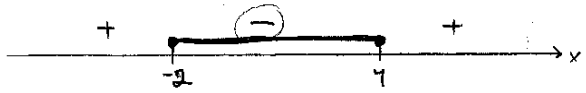
Interval Notation: $(-\infty, +\infty)$

(g) $x^2 - 2x \leq 8$

$x^2 - 2x - 8 \leq 0$ (5)

$(x-4)(x+2) \leq 0$

C.N: $x = 4$, $x = -2$



Interval Notation: $[-2, 4]$

(h) $\dfrac{-2x^2(x+1)^3}{(x-1)^4(x-2)} \leq 0$ (6)

C.N: $x = 0, -1, 1, 2$

Interval Notation: $(-\infty, -1] \cup \{0\} \cup (2, \infty)$

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(4) . $A(2,-3)$ $B(0,2)$

(a) $d_{\overline{AB}} = \sqrt{(2-0)^2 + (-3-2)^2}$ (2)

$= \sqrt{4+25}$

$d_{\overline{AB}} = \sqrt{29}$

(b) $M\left(\frac{2+0}{2}, \frac{-3+2}{2}\right) \Rightarrow M\left(1, -\frac{1}{2}\right)$ (2)

(c) Midpt is the Center & $r = \frac{1}{2} d_{\overline{AB}} = \frac{1}{2}\sqrt{29}$

$\left(x-1\right)^2 + \left(y+\frac{1}{2}\right)^2 = \frac{29}{4}$ (5)

(d) $m = \frac{2+3}{0-2} = \frac{5}{-2} \Rightarrow y - 2 = -\frac{5}{2}(x-0)$

$y = -\frac{5}{2}x + 2$ (5)

(5) $2x^2 + 2y^2 + 8x + 7 = 0$

$(2x^2 + 8x) + 2y^2 = -7$

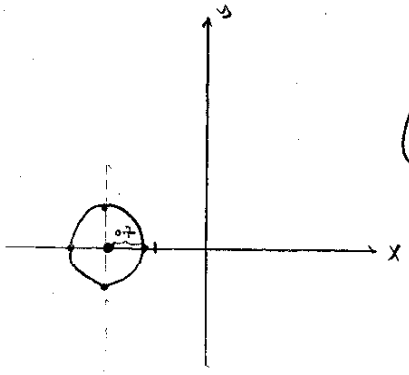
$2(x^2 + 4x\ \ ) + 2y^2 = -7$

$2(x^2 + 4x + 2^2) + 2y^2 = -7 + 2(2^2)$

$2(x+2)^2 + 2y^2 = -7 + 8$

$(x+2)^2 + y^2 = \frac{1}{2}$

$C(-2, 0)$, $r = \sqrt{\frac{1}{2}} \approx 0.7$



(6)

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(6) (a) yes it is a function because it passes the vertical line test. (2)

(b) Domain: $[-3, -2) \cup (2, \infty)$ (2)

(c) Range: $(-\infty, 2)$ (2)

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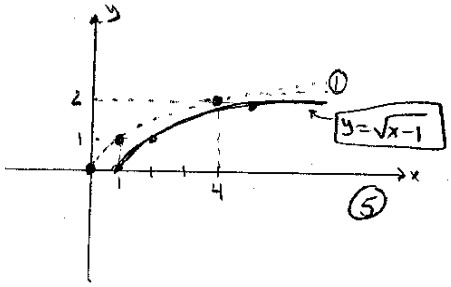
(7) (a) $y = \sqrt{x-1}$

① family fun.: $y = \sqrt{x}$

② $y = \sqrt{x-1}$ shift to right one unit.



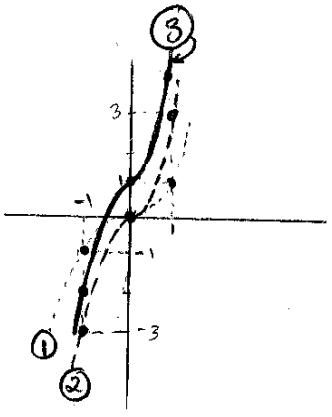
(5)

(b) $y = 2x^3 + 1$

① family: $y = x^3$

② $y = 2x^3$ ⇒ all y-coord. mult. by "3"

③ $y = 2x^3 + 1$ ⇒ shift up 1 unit.



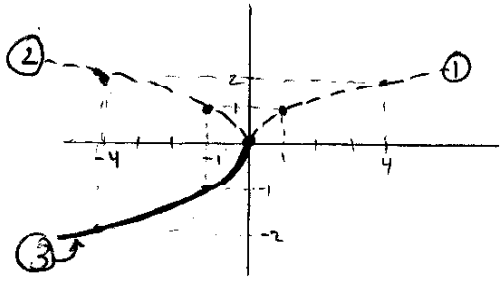
(5)

(c) $y = -\sqrt{-x}$

① family: $y = \sqrt{x}$

② $y = \sqrt{-x}$ all x-coord. mult. by "−1" (reflect across y-axis)

③ $y = -\sqrt{-x}$ all y-coord. Mult. by "(−1)" (reflect across x-axis)



(6)

⑦ continued:

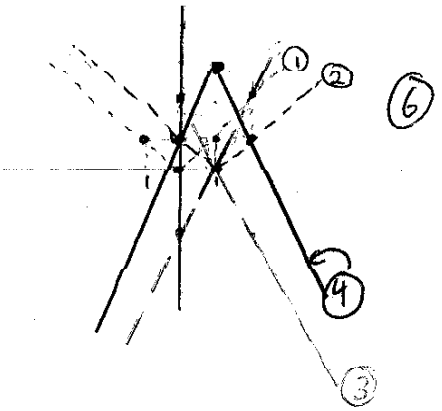
ⓓ $y = -2|x-1|+3$

① family $y = |x|$

② $y = |x-1|$ shift right "1" unit.

③ $y = -2|x-1|$ Mult. all y-coord. by "2".

④ $y = 2|x-1|+3$ shift up "3" units.



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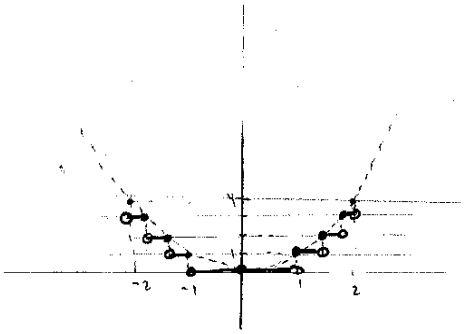
⑧ (Extra Credit): $y = \| x^2 \|$



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